A STUDY OF ENERGY AND ITS IMPLICATIONS

SPACE TEMPORAL STRUCTURE OF ENERGY

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"Time is considered like a flow, as a growing magnitude that advances inexorably carrying every possible event, disappearing its trace and assuring its continuity through the predictable future. But past and future are not more than unreal forms, so are they undisputed proofs of the time existence? Can be past and future like energetic instant states inside a characteristic process of energetic variation that we observe as "time passage"?"

The development of the initial equations shows the magnitude "space" like a mathematical way that possesses all the necessary characteristics for the development and formation of any observable real process. The vacuum, defined mathematically through the magnitude space characterized by the equation "$s=(G\varepsilon\mu)\cdot m$", allows itself the existence of processess with energetic transmission as the electromagnetic radiation, the formation of matter defined by some properties as mass and electrical charge across the spatial curvature, and assures the constancy of light velocity with independence from the velocity of the reference system.

The evolution of the initial hypotheses bears to the known consequences in development of the special relativity theory and relates general relativity with the elemental particles across the spatial curvature and its relation with a constant force value $F=c^4/G$ and Planck space, time, mass and energy values, all implied in the inertial feature so called mass and also in the electrical charge property. The constant relation $E/m=c^2$ is associated to volume through the above mentioned constant force and tension, and the values already defined by Planck appear interpreted as a "coupling" between "x" and Compton's wave length "λ" according to the deduced equation $\lambda\cdot x=G\cdot h/c^3$.

"...The photon would remain trapped by the central punctual mass, only to distance whose value will be the associated space to mass: $r=x=mG/c^2$, which is independent of the according to Compton formula photon mass."

"...we might consider their spatial magnitudes by equivalence and so a spatial curvature with these characteristics would behave like a photon trapped to distance r = x from the center of a sphere with that radio, in which every points which conform it to the same distance show a tension $T`=c^4/G$. And mass attribute appears naturally.

"…*mass appears from the coupling mass value (coincident with Planck mass) modulated by the interaction between two forces*."

"…***proper time appears as a function of temperature and entropy variation***, and second one, in natural ***processes transmitted at light velocity, proper time does not exist.***"

"…But we need to employ time to predict natural processes we observe, considering it consequently an undisputed probe about the existence of time. In this way *we frame a useful entelechy that only exits from the related to the observer system point of view to study other inertial or non inertial external systems, where time does not exist or it is only an equivalence with energy exchange.*

Even on inertial physical systems with non entropy variations and constant velocity, equivalent to resting systems, or on non inertial systems where acceleration exists, with equivalences on gravity or weightlessness depending on direction of the acceleration, time does not exist, although an external observer can use the concept of time to define those frames. And in turn, one observer placed on a system for which proper time does not exist, could use time to define each other observed external system.
It is only on frames where an entropy variation occurs, where we can devise the proper time concept, *but considering distinct signs depending on signs of the entropic variation.*"

"*...Life is a feedback reaction that permits to bear its increasing of entropy which tends to destroy the transmission of information over time, first by using energy to control that entropy variation, and as a certain degradation limit, by using energy to be opportunely renewed into another related system achieving the survival of the information.* **Information fights unconsciously to be transmitted and not be destroyed by probability.**"

This article is based on the consideration of mathematical premises that are developed up to stating a series of physical consequences, and they affirm the existing interdependence between both properties, starting from the existing coherence in the relation of the magnitudes time-energy, by integrating the Universal Gravitation constant as an essential part.

The conceptual premises of this theory consider three basic questions that should complement those of mathematical kind:

1.- Time is only the unreality form which defines that in processes in which an energy interchange exists, for an energy variation the energetic condition "E_2" shall not be able to exist without the energy condition "E_1", both of them limited by the impossibility of juxtaposition. The known Universe is just a heap of instantaneous energetic states which are not capable of juxtaposition. Time is equivalent to energy variation.
2.– Time and Gravity are related across the universal gravitation constant "G".
3.- Time is a feature from a mathematical medium (magnitude space), it is another property that defines it, time itself is implied in the variation of instantaneous energetic states characterized by its probabilistic state, that is, for its entropy. The above mentioned magnitude shall have to be related to the entropy variation.

From Relativity, we know that time is not absolute, but my question advances still a little more and makes the question if time always exists in all the possible situations, if it is relative for other different situations of the movement state and of gravity, and if time advances or it is only an appearance.

1.- VALIDIDITY OF DIFFERENTIALS EQUATIONS FOR ENERGY AND MASS FUNCTIONS.

I propose next differential relations among energy, mass and time, where "G" correspond to the universal gravitational constant and "c" is the velocity of light in vacuum.

$$dE = \frac{G}{c}d\left(\frac{m^2}{t}\right) \;;\quad dm = \frac{c^3}{G}dt$$

Integrating by parts initial equations, we obtain the energy form that presents two integration constants denominated "C_m" and "C_e".

$$\int dE = \frac{G}{c}\left(\int u\cdot dv + \int v\cdot du\right) \quad;\dots\quad \int dE = \frac{G}{c}\left(\frac{-c^6}{G^2}t + \frac{C_m^{\,2}}{t} - \frac{2c^3C_m}{G}\ln t + \frac{2c^6t}{G^2} + \frac{2c^3C_m}{G}\ln t\right) + C_e \;;$$

$$E = \frac{c^5 t}{G} + \frac{C_m^2 G}{ct} + C_e \qquad \textbf{[1.1]}$$

From [1.1] and mass equation "$m = \frac{c^3 t}{G} + C_m$" we obtain the most generic term of energy for a resting particle with mass "m":

$$E = (m - C_m)c^2 + \frac{C_m^2}{m - C_m}c^2 + C_e \quad \textbf{[1.2]} \quad \text{Resting energy equation.}$$

(We can rewrite this equation in the form: $\Delta E = \Delta m \cdot c^2 + \frac{C_m^2}{\Delta m}c^2 \quad \textbf{[1.3]}$, considering "$\Delta E = E - C_e$", "$\Delta m = m - C_m$" and "$C_m$", "$C_e$" been two integration constants).

From the general energy equation, [1.2] can simplify for different constants limits, and simplifies to the Einstein energy-mass correspondence for "$C_m = 0$, $C_e = 0$" values:

I.- $C_m = 0$; $C_e = 0$ $\quad \Rightarrow \quad$ $E = mc^2$

II.- $C_m = 0$ $\quad \Rightarrow \quad$ $E = mc^2 + C_e$

III.- $C_e = 0$ $\quad \Rightarrow \quad$ $E = \frac{(m - C_m)^2 + C_m^2}{m - C_m}c^2$

We can observe maximum and minimum values by deriving energy, considering "C_m" a constant value:

1. $\frac{dE}{dm} = 0$ *implies two solutions:* $m = \{0, 2C_m\}$ $\qquad / \qquad m = 2C_m \equiv \Delta m = C_m$

2. $\frac{d^2 E}{dm^2} = \frac{2C_m^2 c^2}{m^3} \quad \Rightarrow \quad \frac{d^2 E}{dm^2} = \frac{2C_m^2 c^2}{\Delta m^3} = \begin{cases} \dfrac{2C_m^2 c^2}{C_m^3} = \dfrac{2c^2}{C_m} > 0 \text{ minimum} \Leftrightarrow C_m > 0. \\[4mm] \dfrac{2C_m^2 c^2}{(m^{\to 0} - C_m)^3} = \dfrac{-2c^2}{C_m} < 0 \Leftrightarrow C_m > 0. \end{cases}$

From [1.3], $\quad m = 2C_m$, $\Delta m = C_m \Rightarrow \Delta E_{min.} = 2C_m c^2 \quad ; \quad \Delta m = \frac{c^3 t}{G} \Rightarrow t_{\Delta E min.} = \frac{C_m G}{c^3}$

$\qquad\qquad m = 0 \qquad\qquad \Rightarrow \Delta E_{max.} = -2C_m c^2 \quad ; \quad -C_m = \frac{c^3 t}{G} \Rightarrow t_{\Delta E max.} = \frac{-C_m G}{c^3}$

$\left. \begin{aligned} \Delta E_{min.} &= 2C_m c^2 \\ t_{\Delta E min} &= \frac{C_m G}{c^3} \end{aligned} \right\} \quad \Delta E_{min.} = \frac{2c^5 t_{\Delta E min}}{G} \quad ; \quad \Delta E_{min.} = 2C_m c^2 = 2\Delta m_{\Delta E min} c^2 ; \quad \underline{\Delta m_{\Delta E min} = \frac{c^3}{G} t_{\Delta E min.}}$

$\frac{t_{\Delta E min}}{C_m} = 2{,}476 \cdot 10^{-36} \left[\frac{s}{kg}\right] \quad ; \quad \frac{\Delta E_{min}}{C_m} = 1{,}122 \cdot 10^{30} \frac{Mev}{kg}$

A.) Taking "C_m" as a constant value and *allowing for* "$C_e = C_m \cdot c^2$", "$t_{\Delta e min}$" would take the form "$C_e G/c^5$", and the resting energy of a mass "m" would complies with "$m_{Emin} = 2C_m$" so that, "$E - C_e = 2C_m$, $E_{min.} = 3C_m$".
Resting energy so that "$C_m \to 0$" or "$m - C_m \approx m$" implies again the approximation "$E \approx m \cdot c^2$").

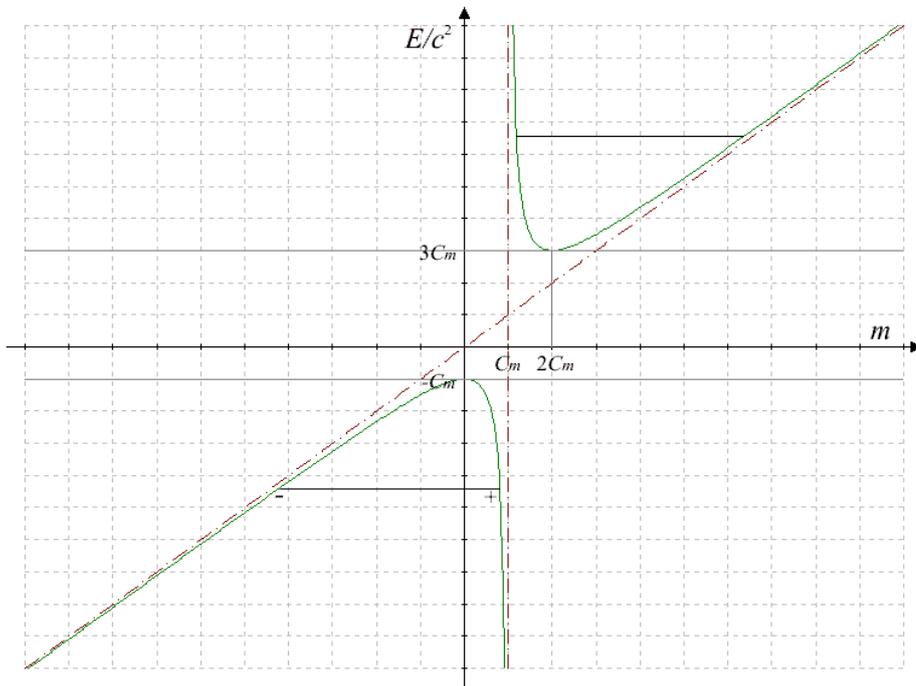$$E = \frac{(m - C_m)^2 + mC_m}{m - C_m} c^2 \qquad \textbf{[1.4]}$$

Figure 1.1. Resting energy of mass. Equation [1.4] graphical representation.
Energy function "$E=f(m)$" so that "$C_m \rightarrow 0$" or masses high enough, presents a right enough approximation to "$E=m \cdot c^2$", drawn by a straight line on the graphical representation. Otherwise an asymmetrical form is observed, and different values of mass can possess the same value of energy.

"C_m" should not acquire any value and it should have a very small concrete value in accordance with experience.

From figure 1.1., two totally different cases of the energy value are observed:

1.1.- **$m>C_m$**. For high enough masses energy approximates to "$E=m \cdot c^2$" but for masses closed to "C_m" energy would not be a valid relation. As well energy diminishes when mass do it within interval $[C_m, 2C_m]$, and increases for "$m>2C_m$".
The approaching to value "$m=C_m$" implies that energy increases itself exponentially tending to infinite.
2.1.– **$0 \leq m < C_m$**. It is observed that value "$m=0$" can only exist from negative energy values, unless "C_m" takes a negative significance. For "$m=0$", energy shows a maximum point, from which it will diminish to $-\infty$ once mass is near to "C_m" value.
2.2.- **$m<0$**. For negative enough values energy complies with "$E=m \cdot c^2$" approximation.

B.) Without a special consideration for the energy integration constant "C_e", we can study the energy-mass behaviour from the general equation [1.3].
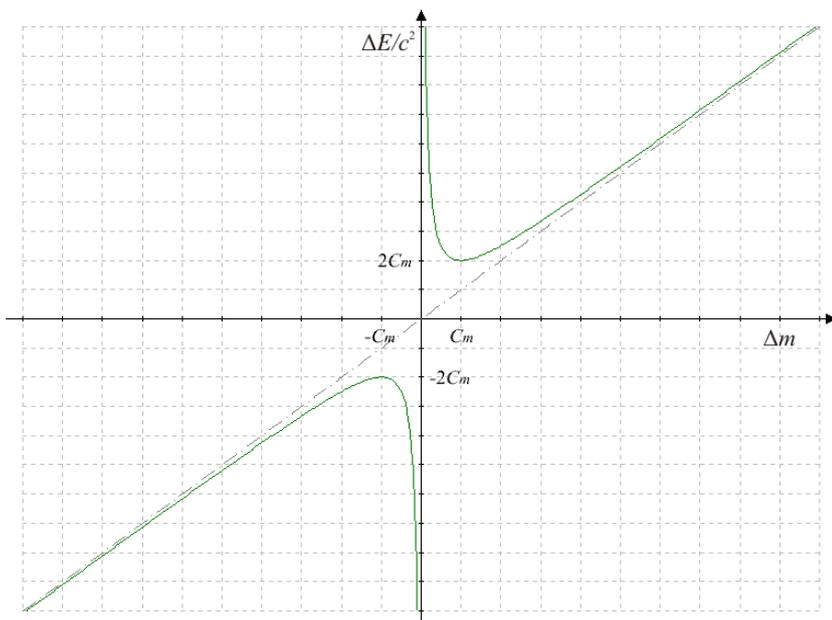

Figure 1.2. Graphical representation for general equation [1.3].
Representation of energy variation "$E-C_e$" versus mass variation "$m-C_m$" presents next physical peculiarities: asymptote in "$\Delta m=0$ $(m=C_m)$"; minimum energy variation value "$\Delta E=2C_m \cdot c^2$" for "$\Delta m=C_m$ $(m=2C_m)$" ; maximum value "$\Delta E=-2C_m \cdot c^2$" for "$\Delta m=-C_m$ $(m=0)$". Likewise, for masses variation high enough, small enough negatives values and "C_m" values nearly closed to zero imply the approximation "$\Delta m \approx m$" and the term "$\Delta E \approx \Delta m \cdot c^2$".

In scales of mass variation nearby "$\pm C_m$" the value of energy variation deviates from "$\Delta m \cdot c^2$" so that if we continue diminishing or "increasing" from $\Delta m = \pm C_m$, *energy variation tends respectively to "$\pm\infty$"*.

Studying function [1.1] $E=f(t)$, in the same way as $E=f(m)$, we can observe two solutions for each energy value, excepting for maximum and minimum ones.

From [1.1], time-energy function presents the form: $t = \dfrac{G}{2c^5}\left(\Delta E \pm \sqrt{\Delta E^2 - 4C_m{}^2 c^4}\right)$ **[1.5]**

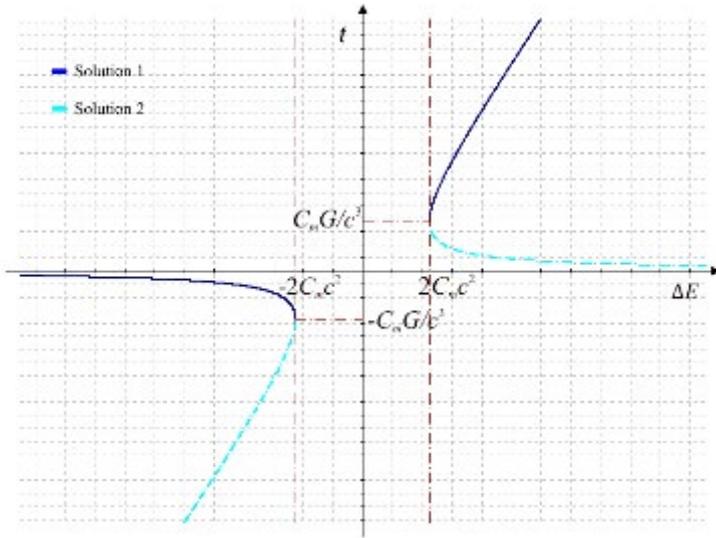considering "$\Delta E = E - C_e$" and "C_e" the integration constant appeared in equation [1.1].

Figure 1.3. Graphical representation for equation [1.5]. Time presents an asymmetry with regard to energy for each one of the two solutions, presenting imaginary values within interval $E-C_e \in (-2C_m c^2, 2C_m c^2)$.

The study of time-energy relation bears to a series of conclusions, between which that *in a process with an energy interchanging equal to zero, time does not exist*:

1.- From [1.1], integration constant "C_m" form is: $C_m = \sqrt{\dfrac{ct}{G}\left(\Delta E - \dfrac{c^5 t}{G}\right)}$ and to exclude an imaginary solution must be complied:

$\Delta E \geq \dfrac{c^5 t}{G}$; $\Delta E \geq \dfrac{c^4 x}{G}$ $/\ t \neq 0$ *and taking* $"x = c \cdot t"$ *as the associated space.*

2.- Alternatively:

$C_m = \sqrt{\dfrac{ct}{G}\left(\Delta E - \Delta mc^2\right)}$; $\Delta E \geq \Delta m \cdot c^2$; $E - mc^2 \geq C_e - C_m c^2$.

3.- From [1.5], $t = f(E)$ $\Rightarrow$ $C_m \to 0$, $t = \left\{\dfrac{G\Delta E}{c^5}, 0\right\}$

Excluding again time imaginary values: $\Delta E \geq 2C_m c^2$, $\Delta E \leq -2C_m c^2$.

4.- $\Delta E = 0$ implies an imaginary time value: "$t = \pm \dfrac{C_m G}{t} i$ ".

2.- HIGH MASSES CONSIDERATION.

For high masses consideration, we can consider the approximation "$E = m \cdot c^2$" whenever the integration constants have sufficiently small values, and light velocity in vacuum would present next definitions:

$c = \sqrt[3]{G\dfrac{m}{t}}$, $c = \sqrt[5]{G\dfrac{E}{t}}$

Being "t" the "associated time of the particle" and associating this attribute to energy, it shall comply with "$E = c^5 t/G$". It is observed:

1.- Energy and time are equivalent.

2.- Every point of the Universe would comply with relation "$E/t = c^5/G$ / $C_m \approx 0$".

3.- Light velocity is the logical consequence that those relations should be fulfilled in any point of the Universe. If occasionally the relation $E/t=c^5/G$ were changed in vacuum, light velocity would vary. In fact, that relation varies depending on the propagation medium.

In accordance with equations, time depends on energy variation by the next mean:
$\Delta E>0 \rightarrow \Delta t>0$.
$\Delta E=0 \rightarrow \Delta t=0$.
$\Delta E<0 \rightarrow \Delta t<0$ (*time in debt*).

Supposing a particle with mass *"m"* and velocity *"v"* such as *"v<<c"*, we can apply previous equations to study force implied.

1. $E = \frac{1}{2}mv^2 + U$; $\quad U = \frac{c^5 t}{G} - \frac{1}{2}\frac{c^3 t}{G}v^2$; $\quad U = \frac{c^3 t}{G}\left(c^2 - \frac{v^2}{2}\right)$ **[2.1]** $\Rightarrow U = m\left(c^2 - \frac{v^2}{2}\right)$ **[2.2]**

$v=0 \Rightarrow U = mc^2 = \frac{c^5 t}{G}$ $\qquad ; \qquad$ $v=c \Rightarrow U = \frac{mc^2}{2} = \frac{c^5 t}{2G}$

2. $\left. \begin{aligned} E &= \frac{1}{2}mv^2 + U \\[2mm] E &= \frac{G}{c}\frac{m^2}{t} \end{aligned} \right\}$ $\qquad U = \frac{2Gm^2 - mv^2 ct}{2ct}$ **[2.3]**

Above equations define potential energy. According to [2.1] and [2.3], they depend on time value, specifically on "particle time, *t*", already defined previously. I will consider now a spatial magnitude *"x"* defined as the created or necessary space to becoming the existence of time particle complying with *"x=c·t"*.

From [2.1] , $\quad U = \frac{c^4}{G}x - \frac{c^2}{2G}xv^2$ $\qquad , \quad \left\{ \begin{aligned} v=0 &\Rightarrow \quad U = \frac{c^4}{G}x \\[2mm] v=c &\Rightarrow \quad U = \frac{c^4}{2G}x \end{aligned} \right.$

From [2.3], and considering *"x=ct"*: $\quad U = G\frac{m^2}{x} - \frac{1}{2}mv^2$, $\quad \begin{aligned} v=0 &\Rightarrow \quad U = G\frac{m^2}{x} \\[2mm] v=c &\Rightarrow \quad U = G\frac{m^2}{x} - \frac{1}{2}m\,c^2 \end{aligned}$

"*U*" defines the potential energy of a mass "*m*" that carries an associate magnitude space created to the vacuum light velocity during the associated time to the particle. It coincides with the classical value of the gravitatory potential energy, at least in its shape.

By considering: $\vec{F} = -\vec{\nabla}U$.

- $U = G\frac{m^2}{x} - \frac{1}{2}mv^2$; $\quad F = -\frac{\partial U}{\partial x}$; $\quad \boxed{F = G\frac{m^2}{x^2} + \left(\frac{v^2}{2} - 2G\frac{m}{x}\right)\frac{\partial m}{\partial x} + m\frac{dv}{dt}}$ **[2.4]**

- $U = \frac{c^4}{G}x - \frac{c^2}{2G}v^2 x$; $\quad F = \frac{c^2}{2G}v^2 + \frac{c^2}{G}x\frac{dv}{dt} - \frac{c^4}{G}$ **[2.5]**

3.- GENERALIZATION OF FORCE IN ITS RELATIONSHIP WITH ENERGY, MASS AND ASSOCIATED ESPACE "*x*".

The expression of force [2.5] is shaped by three force terms in the form "$F=F_1+F_2-F_3$" and has been developed for values of velocities near to zero. *Generalizing for any value of velocity*:

$$F_1 = \frac{c^2}{2G} v^2$$

$$F_1 = F\gamma (1 \pm x)^{\pm 1/2} \qquad / \qquad F\gamma = \frac{c^2}{G}\xi \quad ; \quad x = v^2\varepsilon$$

$$\xi = v^2 \quad ; \quad \xi \cdot \varepsilon = 1 \quad \Rightarrow \quad \varepsilon = \frac{1}{v^2} \quad ; \quad x = 1.$$

$$\xi = c^2 \quad ; \quad \xi \cdot \varepsilon = 1 \quad \Rightarrow \quad \varepsilon = \frac{1}{c^2} \quad ; \quad x = \frac{v^2}{c^2}.$$

$$F_1 = \frac{c^4}{G}\left(1 \pm \frac{v^2}{c^2}\right)^{\pm 1/2} \qquad ; \quad 1.)\ v > ic \quad ; \quad 2.)\ v = c \Rightarrow E = 0 \quad ; \quad 3.)\ v \geq ic \quad ; \quad 4.)\ v \leq c$$

From point 4 appears: " $F_1 = \dfrac{c^4}{G\sqrt{1 - \dfrac{v^2}{c^2}}}$ " and equation [2.5] is transformed to:

$$\boxed{F = \frac{c^4}{G}\left(\frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} - 1\right) + m\frac{dv}{dt}} \quad \textbf{[3.1]}, \quad \text{including a kinetical expression: } F_{kinetic} = \frac{c^4}{G}\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)$$

and the value of the generalization in the force related to the magnitudes energy and mass is:

$$F_{E,m} = \frac{c^4}{G\sqrt{1 - \dfrac{v^2}{c^2}}} \qquad \textbf{[3.2.]}$$

That simplify to " $F_{E,m} = c^4/G$ " for " $v \ll c$ " values.

$$W = \int_0^x F_{E,m}\, dx \quad . \text{ For each velocity value, } \quad W = \frac{c^4 x}{G\sqrt{1 - v^2/c^2}} \qquad \textbf{[3.3]} \qquad W = E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

The relativity equation of Energy appears as a result of the work produced by a conservative strength whose value is proportional to " c^4/G ".

$$\left.\begin{array}{l} E = mc^2 = \dfrac{m_0 c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}} \\[2em] E = \dfrac{G}{c}\dfrac{m^2}{t} \ , \quad t = \dfrac{G}{c^3}m \Leftrightarrow C_m \to 0 \end{array}\right\} \qquad t = \frac{m_0 G}{c^3 \sqrt{1 - v^2/c^2}} \quad \textbf{[3.4]} \ ; \ t = \frac{t_0}{\sqrt{1 - v^2/c^2}} \ \ \begin{array}{l}\textit{Relativistic}\\ \textit{equation of time.}\end{array}$$

Remembering the equation " $c = \sqrt[5]{G\dfrac{E}{t}}$ ", $\qquad c = \sqrt[5]{G\dfrac{E_0\sqrt{1 - v^2/c^2}}{t_0\sqrt{1 - v^2/c^2}}} \quad \Rightarrow \quad c = \sqrt[5]{G\dfrac{E_0}{t_0}} \neq f(v)$

"c" appears as a constant one that besides, it is not function of the reference system velocity.

$$\left.\begin{array}{l} E = \dfrac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \\[2em] E = \dfrac{G}{c}\dfrac{m^2}{t} \end{array}\right\} \qquad m = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} \qquad \textit{Relativistic equation of mass.}$$

4.- STUDY OF THE VELOCITY-TIME FUNCTION.

Previous point assigns a velocity-time relation according to expression [3.4]: $v^2 = c^2 - \dfrac{G^2}{c^4}\dfrac{m_0^2}{t^2}$, *where the necessary*

condition so that velocity does not present a complex solution is: $t \geq \dfrac{m_0 G}{c^3}$.

Equation [3.4] can be expressed in binomial form "$v = c(1-x)^{\frac{1}{2}}$" and developed consequently:

$$v = c\left(1 - \frac{1}{2}\frac{G^2}{c^6}\frac{m_0^2}{t^2} - \frac{1}{8}\frac{G^4}{c^{12}}\frac{m_0^4}{t^4} - \cdots\right)$$

Using approximation "$v \gg 0$", "$v = c - \dfrac{k}{t^2}$" with limits "$\lim\limits_{t \to 0} v = -\infty, \ \lim\limits_{t \to \infty} v = c$",

the consequence is: $\qquad \dfrac{t_0^2}{t^2} = 2\left(1 - \dfrac{v}{c}\right)$

And with a more complete approximation: $\qquad \dfrac{v}{c} = 1 - \dfrac{1}{2}\dfrac{t_0^2}{t^2} - \dfrac{1}{8}\dfrac{t_0^4}{t^4} \quad / \ t_0 = \dfrac{m_0 \cdot G}{c^3}$

Making use of time-proper mass relation:

$$t = \frac{m_0 G}{c^3\sqrt{1 - \frac{v^2}{c^2}}} \quad ; \quad t = \frac{m_0 G}{c^3}\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + ..\right) \ / \text{ approximating to 2nd degree } ; \quad t = \frac{m_0 G}{c^3} + \frac{1}{2}\frac{m_0 G}{c^5}v^2$$

So that "$v = 0$" $\Rightarrow t = t_0 = \dfrac{m_0 G}{c^3} \qquad / \qquad t = t_0 + \dfrac{1}{2}t_0\dfrac{v^2}{c^2}$

$$\boxed{t = t_0 + \frac{1}{2}m_0 v^2\left(\frac{G}{c^5}\right)} \quad \text{[4.1]}$$

In the same way, if it comes closer,

$E = \dfrac{c^5 t_0}{G}\left(1 - \dfrac{v^2}{c^2}\right)^{-\frac{1}{2}}$. 2nd degree aproximation implies: $E = \dfrac{c^5 t_0}{G} + \dfrac{1}{2}\dfrac{c^3 t_0}{G}v^2$; "$E = E_0 + \dfrac{1}{2}m_0 v^2$", being the relativistic

equations: $\qquad \dfrac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0 c^2 + E_c$; $\qquad \dfrac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = t_0 + E_c\left(\dfrac{G}{c^5}\right)$

In short, for "$v \ll c$"	**Relativistic form**
$E = E_0 + \dfrac{1}{2}m_0 v^2$	$\dfrac{m_0 c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}} = m_0 c^2 + E_c$
$t = t_0 + \dfrac{1}{2}m_0 v^2\left(\dfrac{G}{c^5}\right)$	$\dfrac{t_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} = t_0 + E_c\left(\dfrac{G}{c^5}\right)$
$\dfrac{mG}{c^3} = \dfrac{m_0 G}{c^3} + \dfrac{1}{2}m_0 v^2\left(\dfrac{G}{c^5}\right)$	$\dfrac{m_0 G}{c^3\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{m_0 G}{c^3} + E_c\left(\dfrac{G}{c^5}\right)$

5.- TERMODINAMIC STUDY OF THE ASSOCIATED TO ENERGY-MASS PROPER TIME.

According to the 1st and 2nd Termodinamic principles where "dW" is the differential of work carried out by the system, we can consider the value of relativistic energy as a work carried out by the force "$F_{E,m}$". By this way and using previous [3.3] formula, appears a relativistic equation for entropy concept.

$$dQ=dU+dW \quad ; \quad dW=TdS-dU$$

$$dS = \frac{c^4 dx}{T \cdot G \sqrt{1-\frac{v^2}{c^2}}} + \frac{dU}{T} = \frac{dE}{T\sqrt{1-\frac{v^2}{c^2}}} + \frac{dU}{T} \quad ; \quad dS = \frac{1}{T}\left[\frac{c^4 dx}{G\sqrt{1-v^2/c^2}} + \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \right]$$

5.1. In ideal processes in which variation of internal energy is only dependent on variation of temperature, the internal energy does not vary when temperature is constant,

$$dS = \frac{m_0 c^2}{T\sqrt{1-v^2/c^2}} \quad ; \quad dS = \frac{m_0 c^2}{T} \text{ for velocity equal to zero value.}$$

$$dS = \frac{1}{T}\frac{c^5 dt}{G\sqrt{1-v^2/c^2}} \quad ; \quad \boxed{t_0 = \frac{G}{c^5}T\Delta S\sqrt{1-\frac{v^2}{c^2}}} \quad \textbf{[5.1]} \quad \textit{(associated to energy proper time)}.$$

The consequences of this equation are important. First one, ***proper time appears as a function of temperature and entropy variation***, and second one, in natural ***processes transmitted at light velocity, proper time does not exist***.

$$\boxed{t_0 = \left(\frac{G}{c^5}\right)T\Delta S \quad ; \quad t_0 = K \cdot T\Delta S} \quad \textbf{[5.2]} \quad \textit{resting associated proper time}$$

$$\left.\begin{array}{l} t_{01} = K \cdot T\Delta S_1 \\ t_{02} = K \cdot T\Delta S_2 \end{array}\right\} \quad \boxed{\frac{t_1}{t_2} = \frac{\Delta S_1}{\Delta S_2}}$$

5.1.- Generalization in complex systems.

For a particle, the smaller its energy is, the smaller its "associated magnitudes" space and time are.

In this way, relativity in the measurement of time depends no longer on the system speed as well as on gravitational fields just as Einstein´s special and general relativity theories predict, but ***it also would depend on the energetic exchange***.

Generalizing the concept of the equation [5.2], if we were achieving a system with zero Kelvin degrees, time would not exist due to its void value.

Supposing two systems, first one showing an significant energetic exchange (for example an human being in classical temperature conditions) and a second one (for example an human being in low temperatures), in the first case the energetic exchange used to enlarge or diminish the system entropy is bigger than in the second one, (although the final balance in feed-back complex systems is $\Delta S>0$). "Time" in first systems will elapse more quickly than in second ones in which the increasing into the entropy balance will be smaller.

Entropy variations with positive or negative values only means that in an energy variation process, the system goes from the smallest to the biggest or from the biggest to the smallest probability state, according to the Boltzmann´s relation "$S=K \cdot logP$".

Time is for each system a measurement of its entropy variation and temperature attributes, in turn dependent on the produced energetic exchange which can produce positive or negative entropy variation, and thus time can possesses distinct signs (time in debt for $t<0$), keeping in mind that the final entropy balance will depend on the complexity of the system and its energetic exchange, being able to change to another different system of bigger (towards no equilibrium) or smaller orderliness (towards equilibrium) by means of this incessant exchange of energy that our Universe seems to be.

In this way, and letting myself to argue about something so complex as life, it is a feedback reaction that permits to bear its increasing of entropy which tends to destroy the transmission of information over time, first by using energy to

control that entropy variation, and as a certain degradation limit, by using energy to be opportunely renewed into another related system achieving the survival of the information. **Information fights unconsciously to be transmitted and not be destroyed by probability**.

In relation to Albert Einstein´s remark whose concept was so many times repeated to his friend M. Besso: "the difference among past, present and future is only an illusion", time in its entirety concept does not exist, but there is a continuous energetic flow for each system included in the Universe which depends on its configuration and intimately related to its probabilistic state (according to the equation developed by Boltzmann: *S=K·log P*), so that the entropy variation in the Universe as an isolated system is always bigger than zero.

But we need to employ time to predict natural processes we observe, considering it consequently an undisputed probe about the existence of time. In this way *we frame a useful entelechy that only exits from the related to the observer system point of view to study other inertial or non inertial external systems, where time does not exist or it is only an equivalence with energy exchange.*

Even on inertial physical systems with non entropy variations and constant velocity, equivalent to resting systems, or non inertial systems where acceleration exists, with equivalences on gravity or weightlessness depending on direction of the acceleration, time does not exist, although an external observer can use the concept of time to define those frames. And in turn, one observer placed on a system for which proper time does not exist, could use time to define each other observed external system.

It is only on frames where an entropy variation occurs, where we can devise the proper time concept, *but considering distinct signs depending on signs of the entropic variation.*

6.- COUPLING.

The consequence of equivalences between mass and both spatial magnitudes "*x*","*λ*" implies a constant value for spatial relationships.

$$E = mc^2 \quad ; \quad E = h\nu \quad \rightarrow \quad \lambda = \frac{h}{mc} \quad \text{Compton wawelength}$$

$$m = \frac{c^2 x}{G}$$

$$\boxed{\lambda x = \frac{Gh}{c^3}} \qquad \textbf{[6.1]}$$

$$\frac{1}{T} = \frac{mc^2}{h}$$

$$m = \frac{c^3 t}{G}$$

$$\boxed{\frac{1}{Tt} = \frac{c^5}{Gh}} \qquad \textbf{[6.2]} \qquad \text{T: wavelength period; t: time associated to mass.}$$

Considering a *coupling for both spatial and temporal magnitudes "1/T=1/t" and "λ=x"*, mass, space and time acquire the Planck values.

$$m_c = \sqrt{\frac{ch}{G}} \quad \textit{Coupling mass} \ ; \quad x_c = \sqrt{\frac{Gh}{c^3}} \quad \textit{Coupling space} \ ; \quad t_c = \sqrt{\frac{Gh}{c^5}} \quad \textit{Coupling time}$$

$$m_0 = \sqrt{\frac{ch}{G}} \sqrt{1 - \frac{v^2}{c^2}} \quad ; \quad v = nc \quad ; \quad n = \sqrt{1 - \frac{m_0^2 G}{ch}}$$

$$m_0 \leq \sqrt{\frac{ch}{G}} \qquad ; \qquad x_0 \geq \sqrt{\frac{Gh}{c^3}} \qquad ; \qquad t_0 \geq \sqrt{\frac{Gh}{c^5}} \qquad \text{for coupling condition}$$

Spatial, time coupling determine the maximum mass and the minimum size of a particle as well as the minimum time necessary for the transformation space-mass features.

It is observed that the coupling is an inflexion point:

$$\frac{1}{t} \geq \frac{1}{T} \quad \Rightarrow \quad \frac{c^3}{G}\frac{1}{m} \geq \frac{mc^2}{h} \quad ; \quad m \leq \sqrt{\frac{ch}{G}}$$

$$\frac{1}{t} \leq \frac{1}{T} \quad \Rightarrow \quad m \geq \sqrt{\frac{ch}{G}}$$

$$\Big/ \quad \frac{t}{T} = \frac{Gm^2}{ch} \ ; \qquad \boxed{m = \sqrt{\frac{t}{T}} \cdot m_c} \quad \textbf{[6.3]}$$

7.- STUDY OF ENERGETIC VALUE AND IMPULSE IMPLICATED IN ASSOCIATED MASS AND SPACE RELATIONSHIP REFERING TO COUPLING.

For every relation: $<E, m, x, t>$, $E = F_U \cdot x_c$, $E = \sqrt{\dfrac{c^5 h}{G}}$ / $F_U = \dfrac{c^4}{G}$

Being considered "F_U" as a coupling force or unified force.

It is observed that energy value is in the order of 10^{19} Gev, and this value has been considered as a critic one in the unification of forces, which coincides with the active force playing during the space transformation in its inertial characteristic called mass or electromagnetic radiation ($m_0=0$).

$$W = \int F dx = \sqrt{\dfrac{c^5 h}{G}} = \int P \cdot dV = \dfrac{\rho_0 c^2}{\sqrt{1-\dfrac{v^2}{c^2}}} \cdot V_0 \; ; \; m_c = \sqrt{\dfrac{ch}{G}} \; / \; W_U = m_c c^2 \; ; \; \textit{Unified work}$$

The necessary *impulse* for the transformation between the characteristics "E,m,x" is:

$$I_c = F_U \cdot t_c \quad ; \quad I_c = \sqrt{\dfrac{c^3 h}{G}}$$

$$\boxed{W_U \cdot t_c = h \quad \textbf{[7.1]}}$$

The necessary work for the transformation between the features "E,m,x" applied during the coupling time is the Planck constant: "h".

8.- PHYSICAL INTERPRETATION OF ASSOCIATED SPACE.
8.1.- Study of pressure in an isotropic medium defined by its space-mass equivalence.

Supposing the maximum generalization in equations which express the value of pressure on a surface element:
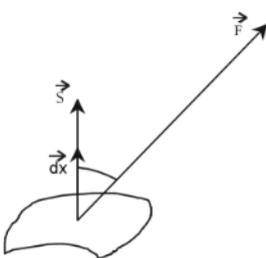
$$\left. \begin{array}{l} P = \dfrac{d\vec{F} \cdot d\vec{x}}{dV} \\[2em] P = \dfrac{\rho_0 c^2}{\sqrt{1-\dfrac{v^2}{c^2}}} \end{array} \right\} \quad dF \cos \alpha = \dfrac{PdV}{dx} = \dfrac{c^4}{G} \dfrac{m_0}{V_0 \sqrt{1-\dfrac{v^2}{c^2}}} \dfrac{dV}{dm} \quad / \quad dm = \dfrac{c^2}{G} dx$$

ρ_0: density at $v=0$.

The necessary conditions to force acquire the unified force value are "$dV=V_0$", "$dm=m_0$":

$$\vec{F} \cdot \vec{u}_x = \dfrac{c^4}{G \sqrt{1-\dfrac{v^2}{c^2}}}$$

It coincides with equation [3.2] and with the first term of equation [2.4] for associated mass, space and time and $\delta m/\delta t=0$, $dv/dt=0$.

□ *The constant relation "$E/m=c^2$" is associated to "dV" with a constant value force.*

Being fulfilled in addition:

$$V = \dfrac{x}{k} \; ; \; V = x \cdot R^2 \quad k: \text{Riemann constant.}$$

FIGURE 8.1.

$$x = m\frac{G}{c^2}\left(\frac{V}{V}\right) \quad / \quad V = x \cdot R^2 \quad \Rightarrow \quad \frac{1}{R^2} = k = \frac{G}{c^2}d$$ "*x*": associated space; "*V*": volume; "*k*": Riemann constant; "*d*": density.

An elementary particle will be equivalent to a curvature of the space with an extraordinary value **k=x/V.**
The associated to energy-mass space can be expressed through Riemann constant, "**x=k·V**".

8.2.- Tension of a string with linear density "*μ=m/L*", on which a wave with speed "*c*" is transmitted.

The equation for tensión in a string with linear density "*μ*" where a wave is transmited with velocity "*v*" presents next form:

$$T' = \mu v^2 = \frac{m}{L}v^2 \qquad ; \qquad v = \sqrt{\frac{T'}{\mu}}$$

FIGURE 8.2. Wave being transmitted with speed "*v*" on a string with tension "*T′* ".

The propagation velocity for an electromagnetic wave is "*c*", and taking a string length "*L*" defined as the associated

space value: $L = x = \dfrac{mG}{c^2}$

$$T' = \mu \cdot c^2 \qquad ; \qquad \boxed{T' = \frac{c^4}{G}}$$

8.3.- Equivalence work-energy-mass.

As it was seen in point 3, the equivalence energy-mass given by Relativity theory, can be defined through the work produced by the action of a force with value "*c⁴/G*" which produces a displacement "*x*" defined as the associated space to mass, whose value is defined by the expression: "*m₀G/c²*".

$$W = \int_0^x F_{E,m} \cdot dx \quad ; \qquad W = \frac{c^4 x}{G\sqrt{1 - \dfrac{v^2}{c^2}}} \qquad ; \qquad W = E = \frac{m_o c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

To accelerate a property mass from "*v = 0*" to "*v = c*" in its associated time, the value of the force involved is:

$$F = m \cdot a = \frac{c^3 t}{G}a = \frac{c^4}{G} \qquad / \qquad a = \frac{c}{t}$$

Using the existing relation between the velocity of a wave in an environment with magnetic permeability "*μ*" and a dielectric constant "*ε*":

$$m = \frac{c^2 x}{G} \quad ; \quad c = \frac{1}{\sqrt{\varepsilon\mu}} \qquad ; \qquad \boldsymbol{x = (G\varepsilon\mu) \cdot m} \qquad \textbf{[8.1]}$$

The space associated to a mass is directly proportional to the same one and to the product of universal gravitation, dielectric and magnetic permeability constants.

8.4.- Application to an inertial system with spherical curvature.
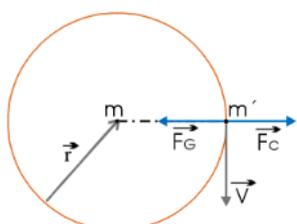
Two punctual masses are supposed, one of which rotates with uniform circular movement around another one.
The gravitatory force has the same value as the centripetal force for k´ system, being configured himself like an inertial system.

FIGURE 8.3.

By using the classical formulas that represents the gravitatory and centripetal forces which acts upon the mass "m′ ",

$$\vec{F} = G \cdot \frac{m \cdot m'}{|\vec{r}|^2} \vec{u}_r = m' \cdot \frac{|\vec{v}|^2}{|\vec{r}|} \vec{u}_r \quad \rightarrow \quad |\vec{r}| = r = G \cdot \frac{m}{v^2} \;\; ; \qquad v = \sqrt{G \cdot \frac{m}{r}}$$

Being a photon that is found trapped in the curvature produced by a mass "m" so that it rotates around it to a distance "r" permitting a situation of equilibrium when it transforms the system defined by photon into an inertial system with constant velocity "c".

$$\Leftrightarrow \quad v_{m'} = c \quad \Rightarrow \quad m = \frac{c^2 \cdot r}{G} \qquad / \qquad r = x \;\; : \; (\text{associated space to mass})$$

The photon would remain trapped by the central punctual mass, only to a distance whose value will be the associated space to mass: $r = x = mG/c^2$, which is independent of the according to Compton formula photon mass.

In fact, if $m = m'$, we might consider their spatial magnitudes by equivalence and so a spatial curvature with these characteristics would behave like a photon trapped to distance $r = x$ from the center of a sphere with that radio, in which every points which conform it to the same distance show a tension $T' = c^4/G$. And mass attribute appears naturally.

This spherical curvature could assume the property $m = c^2 x/G$ and energy $E = W = F \cdot x = c^4 x/G$.

$$|\vec{T'}| = |\vec{F}_G| \quad ; \quad \left.\begin{array}{c} \dfrac{c^4}{G} = G\dfrac{m \cdot m_{photon}}{x^2} \\[2mm] m_{ph} = \dfrac{h}{\lambda c} \;\; / \;\; \lambda = \lambda_{Compton} \end{array}\right\} \quad \begin{array}{c} \lambda = \dfrac{G^2 h}{c^5} \dfrac{m}{x^2} \\[2mm] m = \dfrac{c^2 x}{G} \end{array} \quad ; \quad \lambda x = \dfrac{Gh}{c^3}$$

This equation is the same result as it was already deduced previously from a totally different point of view: See [6.1] relation, λ-x ; T-t.

$$\Leftrightarrow \lambda = x \qquad \Rightarrow \qquad x_c = \sqrt{\frac{Gh}{c^3}}$$

9.-PHYSICAL MEANING OF CHARGE AND MASS MAGNITUDES.
9.1.-Theoretical support.
In relation to the point 8.4, and the equation [8.1] from which the way identified across the spatial property "$x = f(m)$" possesses characteristics associated with the electrical charge "q", by applying the same principle developed:

$$\ldots \frac{1}{4\pi\varepsilon} \frac{q^2}{r^2} = m \frac{c^2}{r} \qquad ; \qquad r = \frac{q^2}{4\pi\varepsilon \cdot mc^2} \quad \textbf{[9.1]}$$

Identical equation to the classic electron radius with only the replacement of the charge "q" for the electron charge "e" and its corresponding mass.
By renaming "$r' = x$" due to the correspondence in this case with spherical properties, three spatial variables appear depending on mass:

$$r = \frac{q^2}{4\pi\varepsilon \cdot mc^2} \quad ; \quad r' = x = \frac{mG}{c^2} \quad ; \quad \lambda = \frac{h}{mc}$$

The following conclusions are observed in case of coincidence among the previous magnitudes:

- $\lambda = r' \quad \Rightarrow \quad m_1 = \sqrt{\dfrac{ch}{G}} = m_c$

- $r = r' \quad \Rightarrow \quad m_2 = \dfrac{q}{\sqrt{4\pi\varepsilon G}} \quad / \quad \dfrac{q}{m} = \sqrt{4\pi\varepsilon G}$

- $\boxed{\lambda = r = r'} \quad \Rightarrow \quad m_1 = m_c \;\; ; \qquad \boxed{q = q_U = \sqrt{4\pi h \sqrt{\dfrac{\varepsilon}{\mu}}}} \quad \textbf{[9.2]}$

Where "q_U" can be called "unified charge", been the result of the three spatial parameters coupling.

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Studying now the combined relations "λ, r" for any resting particles and electron in order to relate its electrical charge,

$$\lambda r = \frac{q^2}{4\pi\varepsilon \cdot mc^2}\frac{h}{mc} \qquad \rightarrow \qquad m_o = \sqrt{\left(\frac{1}{4\pi\varepsilon}\frac{q_U^2}{\lambda^2}\right)\left(\frac{G}{c^4}\right)} \cdot m_c \qquad \textbf{[9.3]}$$

$$(\lambda r)_e = \frac{e^2}{4\pi\varepsilon \cdot m_e c^2}\frac{h}{m_e c} \quad ; \quad m_e = \sqrt{\left(\frac{1}{4\pi\varepsilon}\frac{e^2}{\lambda r}\right)\cdot\frac{G}{c^4}} \cdot m_c \qquad \textbf{[9.4]}$$

And combining both equations,

$$\boxed{e = \sqrt{\frac{r}{\lambda}}\cdot q_U} \qquad \textbf{[9.5]}$$

From [9.5] we observe that electron electrical charge appears from the unified charge, once produced a coupling between the spatial parameters "λ, r".

Equation [9.3] can be expressed by the way,

$$\boxed{m_o = \sqrt{\frac{F_{el}}{F_U}}\cdot m_c} \qquad \textbf{[9.6]}$$

once defined "F_{el}" as an electrical force involved in mass formation, "$F_{el}=f(q_U,\lambda)$". We can see that property *mass appears from the coupling mass value (coincident with Planck mass) modulated by the interaction between two forces*. And remembering the equation [6.3] related to [9.6],

$$m = \sqrt{\frac{t}{T}}\cdot m_c$$

$$m = \sqrt{\frac{F_{el}}{F_U}}\cdot m_c \qquad\qquad \boxed{\lambda = \frac{F_U}{F_{el}}r' = \frac{F_U}{F_{el}}x} \qquad\qquad \boxed{\lambda = \sqrt{\frac{F_U}{F_{el}}}\cdot x_c} \quad \textbf{[9.7]}$$

$$x_c = \sqrt{\frac{Gh}{c^3}} \qquad / \qquad \lambda x = \lambda r' = x_c^2 \qquad\qquad \boxed{r' = \sqrt{\frac{F_{el}}{F_U}}\cdot x_c} \quad \textbf{[9.8]}$$

Any magnitudes mass or space defined "$x \equiv r', \lambda$", come from yours coupling parameters (coincident with Planck magnitudes) modulated by the interaction between two forces, one with electrical character.

The omnipresent force of constant value "c^4/G" appears again in this case by forming part of a non dimensional factor, in such a way that a quantization in the above mentioned factor would involve the quantization of mass, charge, space and time.

9.2.-Quantization of magnitudes.

According to deduced above, let's see the existing relation among the Compton wavelength "λ" for a photon with potential mass "m" identical to a resting mass "m_0", its associated space "x", the coupling space "x_c" and the coupling mass "m_c".

Two existing relations whose values are constant and only dependent of m_c and x_c can be verified: $(\lambda \cdot m_o)$ and (x/m_o).

$$\lambda = \frac{h}{p} \qquad ; \qquad x = \frac{m_0 G}{c^2}$$

$$\lambda = x_c \frac{m_c}{m_0} \qquad\qquad \boxed{\frac{\lambda}{x_c} = \frac{x_c}{x} = \frac{m_c}{m_0}}$$

$$x = x_c \frac{m_0}{m_c}$$

Naming to relations:
$$\frac{\lambda}{x_c} = \frac{x_c}{x} = \xi \quad ; \quad \begin{array}{l} x_c = \xi_1 x \\ \lambda = \xi_2 x_c \\ \xi_1 = \xi_2 \end{array}$$

$$\boxed{\lambda = \xi^2 x} \qquad \boxed{m_0 = \frac{\lambda}{\xi^2}\frac{m_c}{x_c}} \qquad \boxed{\begin{array}{l}\frac{m_c}{m_0} = \frac{\lambda}{x_c} = \frac{x_c}{x} = \xi \\[4pt] \lambda m_0 = x_c m_c \\[4pt] \frac{x}{m_0} = \frac{x_c}{m_c}\end{array}}$$
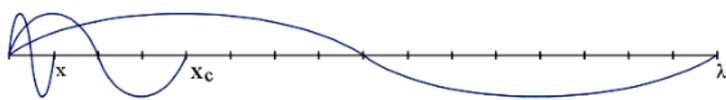
$$m_0 = x\frac{m_c}{x_c}$$

FIGURE 9.1. Graphical representation of x, x_c, λ.

$$\Rightarrow \quad \frac{\lambda}{x} = \frac{x_c \dfrac{m_c}{m_0}}{x_c \dfrac{m_0}{m_c}} = \frac{m_c{}^2}{m_0{}^2}$$

$$\frac{\lambda}{x} = \xi^2$$

$$\boxed{\frac{m_c}{m_0} = \xi} \qquad \textbf{[9.9]}$$

The value "ξ" should introduce quantized values on particles and magnitudes presenting a squared root structure that contain the compromise between one electrical force and other one with unified and constant character.

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