Theoretical Calculation of the Hubble Constant and Relation to CMB and CIB*

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The cosmological displacement of spectral lines observed between different noticeable cosmic objects and Earth depends initially on the relative speeds between them. The present model sets out to evaluate the value of the Hubble constant H_0 by taking account of the behavior of variable electric and magnetic fields, which shapes electromagnetic transmission to produce a spectral red-shift. Application of the model to a sampling of galaxy and quasars leads to a surprising relationship between the CMB, CIB and the cosmological red-shift, as well as to the discrepancy in the conventional distance to quasars

 Keywords: Hubble constant, cosmic microwave background (CMB), cosmic infrared background (CIB), cosmological redshift, galaxy redshift, quasar redshift.

1. Theoretical Introduction

1.1 When propagation of electromagnetic radiation exhibits a spectral line displacement, it is also exhibiting an energy variation. The line displacement z_ℓ and the energy variation ΔE can both be expressed in terms of observed wavelength λ and the emitted wavelength λ_e. The expressions are:

$$z_\ell = (\lambda - \lambda_e)/\lambda_e \quad , \quad \Delta E = -h\Delta\nu = ch\left(1/\lambda_e - 1/\lambda\right) = chz_\ell/\lambda$$

1.2. Let us consider the *variable magnetic and electric fields* component of the electromagnetic wave and the associated frequency of wave transmission ν. The existence of a variable electrical field defines a term in the Ampère-Maxwell Law with magnitude of current intensity:

$$\varepsilon \cdot \frac{d}{dt}\oint_S \mathbf{E} \cdot d\mathbf{S}$$

It is deduced from Gauss's Law by differentiating the electrical charge existing inside a Gauss surface S with respect to time that:

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = q/\varepsilon \quad ; \quad dq/dt = I = \varepsilon \frac{d}{dt}\oint_S \mathbf{E} \cdot d\mathbf{S}$$

So in vacuum, where there are neither charges ($\rho = 0$) nor electrical currents ($\mathbf{j} = 0$), the Ampère-Maxwell Law presents the well-known form:

$$\oint_L \mathbf{B} \cdot d\mathbf{l} = \mu_0\varepsilon_0 \frac{d}{dt}\oint_S \mathbf{E} \cdot d\mathbf{S} \quad .$$

With the above intensity, a photon viewed as a vacuum perturbation will appear analogous to a nanocircuit, which is characterized by frequency ν and self inductance L that acts against the magnetic field change. The existence of L is supported by the *vacuum impedance* $\Omega_0 = \sqrt{\mu_0/\varepsilon_0}$ and a variable magnetic field.

1.3 The energetic change ΔE can be associated with the work done between emission time t_e and reception time t_r, defined as

$$W = \int_{t=t_e}^{t=t_r} I^2 Z \cdot dt$$

where Z is impedance, which can be expressed in terms of ohmic resistance R, inductance X_L, and capacitance X_C, as $Z^2 = R^2 + (X_L - X_C)^2$. In *vacuum*, $R = 0$ and $X_C = 0$,

$$Z_0 = X_L = L_0 2\pi\nu\,,\ \Delta E = \int_0^t (2\pi/\lambda) I_0^2 L_0 c dt = (ch/\lambda) z_\ell \qquad (1.1)$$

$$\Leftrightarrow \ L_0 \ \&\ I_0 = \text{constants},\ Z_\ell = (2\pi L_0 I_0^2 / ch) D$$

where D is the distance covered by the electromagnetic wave in vacuum in the time t: $D = ct$.

2. Calculation of the Hubble Constant H_0

The displacement of a spectral line is defined by Hubble's Law in the form $Z = H_0 D / c$. Applying the result of the expression [1.1] to Hubble's law,

$$L_0 I_0^2 = \hbar H_0 \qquad (2.1)$$

A correspondence is observed between the expression (2.1) and the light quantum energy equation $E = h\nu$, and more concretely with: $2\pi L_o \cdot I_o^2 = E_L$.

If we want to calculate the value of Hubble's constant, we will first have to study the energetic term of Eq. (2.1) and consider some premises:

1) The magnitude $I_0 = \varepsilon_0 \frac{d}{dt}\oint_S \mathbf{E} \cdot d\mathbf{S}$ must be related to inherent vacuum properties.

2) Vacuum self inductance L_0 appears as consequence of the variable magnetic field involved in the electromagnetic transmission, and of the vacuum impedance $\Omega_0 = \sqrt{\mu_0/\varepsilon_0}$.

3) We can apply the energy equation for a harmonic oscillator as it corresponds with the oscillation in an electromagnetic wave and even on emission sources.

$$E_H = (n+\tfrac{1}{2})h\nu \quad \text{for} \quad n = 0,\ 1,\ 2,\ \ldots,\ n \quad .$$

These three premises are developed as follows:

- In the same way as the impedance is assigned to the vacuumas an attribute, we assign the Planck length and the Planck time:

$$\Omega_0 = \sqrt{\mu_0/\varepsilon_0} \quad , \quad x_P = \sqrt{G\hbar/c^3} \quad , \quad t_P = \sqrt{G\hbar/c^5} \quad .$$

- We identify $\Omega_0 = X_L = L_0 2\pi\nu_0 = 2\pi \cdot L_0/t_P$, making the vacuum self-inductance

$$L_0 = \Omega_0 t_P / 2\pi \qquad (2.2)$$

- It is possible to define the electrical charge from $q = \sqrt{4\pi \cdot h \cdot \sqrt{\varepsilon_0/\mu_0}} \cdot \sqrt{r/\lambda} = q_U \cdot \sqrt{r/\lambda}$, by defining $r = q^2/4\pi\varepsilon_0 mc^2$, which for $q = e$ and $m = m_e$ yields the classical radius of the electron, and by taking λ to be the Compton wavelength h/mc. Because the electric charge q_U appears by equating two spatial parameters, it will be named 'Unified charge'. Then:

$$\varepsilon_0 \frac{d}{dt}\oint_S \mathbf{E} \cdot d\mathbf{S} = I_0 = q_U c/\lambda_0 \qquad (2.3)$$

- We evaluate $E_H = (n+\frac{1}{2}) \cdot E_L$. The displacement Z and the constant H_0 then follow from Eqs. (1.1), (2.2), (2.3):

$$Z = (n+\tfrac{1}{2})(\Omega_0 x_P q_U^2 / h\lambda_0^2)D = (n+\tfrac{1}{2})(4\pi x_P/\lambda_0^2)D \qquad (2.4)$$

$$H_0 = (n+\tfrac{1}{2})(\Omega_0 x_P c q_U^2 / h\lambda_o^2) = (n+\tfrac{1}{2})(4\pi x_P c/\lambda_0^2) \qquad (2.5)$$

Now we must only choose the value for λ_0. Let us consider initially that this wavelength could be a collateral effect of the electromagnetic transmission and of the spectral displacement due to the mechanism described above, displacement across the space-time. The most suitable value to be used is the associated to 'black body' wavelength observed as microwave background radiation that floods the Universe.

The Wien law tells us λ_{max} from the relation $\lambda_{max}T = 2.898 \cdot 10^{-3}\,\text{m}\cdot{}^{\circ}\text{K}$ with $T = 2.728^{\circ}K$ [6], which implies $\lambda_{max} = \lambda_0 = 1.0623 \times 10^{-3}\,\text{m}$. We use this value, and consider the possible values for the quantum number n. The calculated value for H_0 is then not unique. The values given in Table 2.1 are expressed in (Km s-1)/Mpc units. The source of constants is CODATA 2002. [7]. Note this H_0 values might vary substantially because of possible new measurements in CMB temperature, and slightly due to revision of constant's values. In any event, two adjacent points present a constant difference of 4,17 (Km·s-1)/Mpc.

Table 2.1

n	H_0(km sec^{-1}/Mpc)	n	H_0(km sec^{-1}/Mpc)
0	2,087	**14**	**60,512**
1	6,260	**15**	**64,685**
2	10,433	**16**	**68,858**
3	14,606	**17**	**73,031**
4	18,779	**18**	**77,204**
5	22,953	**19**	**81,378**
6	27,126	20	85,551
7	31,299	21	89,724
8	35,472	22	93,897
9	39,645	23	98,070
10	43,819	24	102,244
11	47,992	25	106,417
12	52,165	26	110,590
13	**56,338**	27	114,763

The values calculated for $n = 13 \ldots 19$ are highlighted. This is because red-shift periodicities have been observed that correspond well with the hydrogen atom quantization in orbitals for the interval $n = 13$ to $n = 19$. Using Equation (2.5) for this interval, $H_0 \in [56.34, 81.38]$ (Km·s-1)/Mpc (limited by dashed lines in Fig. 2.1) that are in perfect accord with the most accurate experimental values measured in recent times. Observe the *wide concordance* with data tested in year 2000 and later.

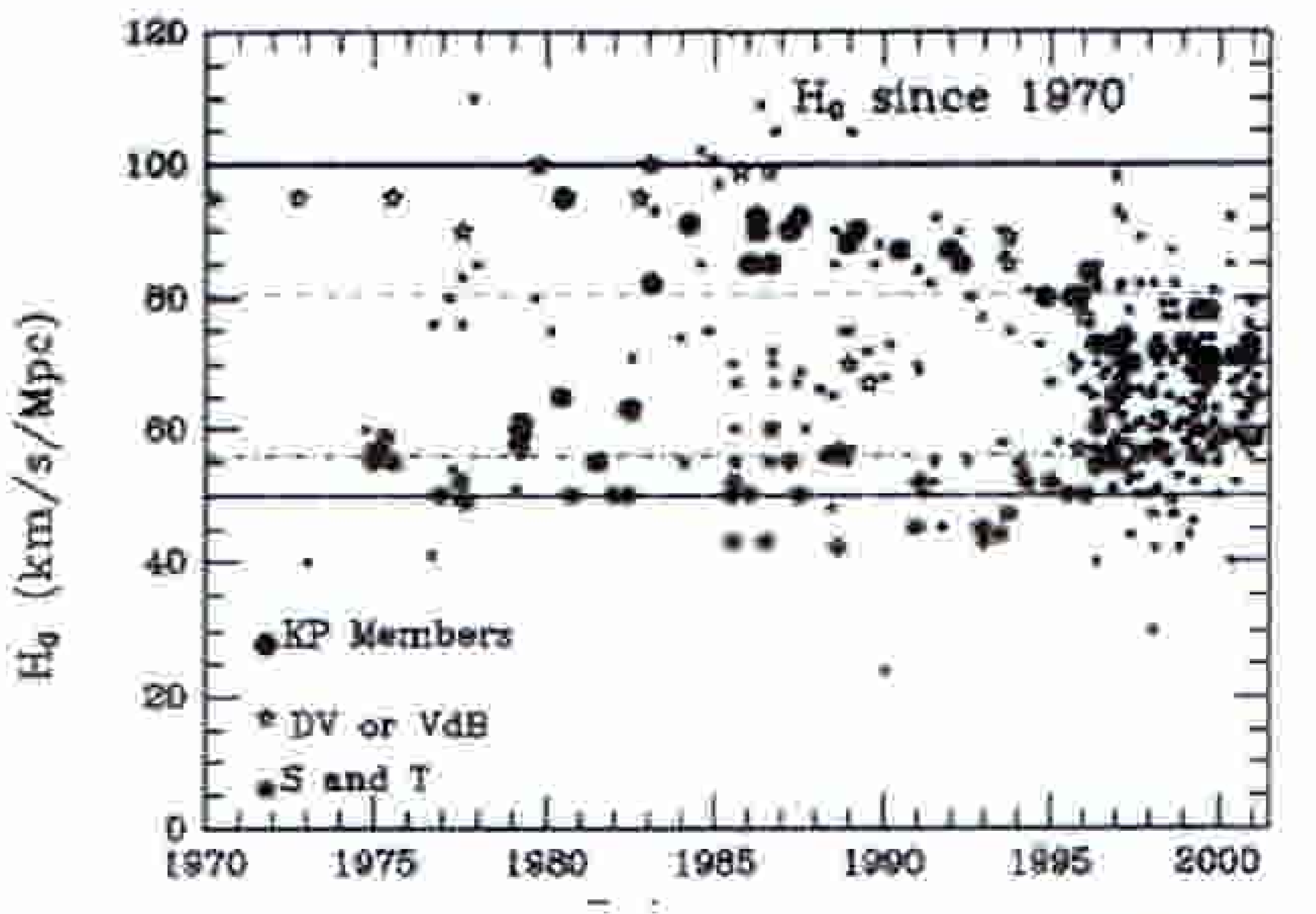

Figure 2.1.

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3. Applications

The self-inductive model has been developed for any source emiting electromagnetic radiation into the vacuum. Let us now apply it to cosmic-size sources, galaxies and quasars, to seek possible differences in the value of Hubble's constant, and in case of confirmed differences, to extract an underlying distinctive parameter. Some authors support the linking of certain quasars to galaxies in contradiction to the believed distances to the quasars. Their hypothesis provides a good tool for researching a possible distinctive parameter, which must then be corroborated by experimental observations.

TABLE 3.1.	EXPERIMENTAL REDSHIFTS. DISTRIBUTION [3]						Equation (3.1)
	GALAXY	z_1	QUASAR	z_2	z_1/z_2	Δz	λ_{02} (μm)
1	NGC 622	0,018	UB1	0,91	0,0198	0,89	149,4
2			BSO1	1,46	0,0123	1,44	117,95
3	NGC 470	0,009	68	1,88	0,0048	1,87	73,5
4			68D	1,53	0,0059	1,52	81,47
5	NGC 1073	0,004	BSO1	1,94	0,0021	1,94	48,24
6			BSO2	0,60	0,0067	0,60	86,74
7			RSO	1,40	0,0029	1,396	56,78
8	NGC 3842	0,02	QSO1	0,34	0,0588	0,320	257,65
9			QSO2	0,95	0,0211	0,93	154,13
10			QSO3	2,20	0,0091	2,16	101,29
11	NGC 4319	0,0057	MARK 205	0,07	0,0814	0,06	303,13
12	MCG03-34-85	0,018	PKS-1327-206	1,17	0,0154	1,15	131,76
13	NGC 5296	0,0083	BSO#1	0,96	0,0086	0,95	98,78
14	3C 303	0.14	UVHC	1.57	0.0892	1.43	317.22
15	1C 2402	0.0667	0844+31	1.83	0.0364	1.76	202.81
16	0924+30	0.0267	COMPACT SOURCE	2.02	0.0132	1.99	122.13
17	NGC 520	0.00758	-	0.77	0.0098	0.76	105.40
18			-	0.67	0.0113	0.66	112.99
19			-	2.11	0.0036	2.10	63.67
20			-	0.72	0.0105	0.71	109.00
					$(z_1/z_2)_{max}$ 0,089		Average 134,70 μm

Consider the redshift information in Table 3.1, taking into account the probability of association [3]. Applying the model with the association of the parameters 'D: distance covered by the electromagnetic wave', 'n, quantum number of the harmonic oscillator', and 'z_i experimental redshift' to each source yields:

1) Galaxy: D_1; n_1; z_1; $\lambda_0 = \lambda_{01} = \lambda_{max}$. CMB $= 1.0623 \cdot 10^{-3}$ m.

2) Quasar: D_2; n_2; z_2; $\lambda_0 = \lambda_{02}$.

Application of $D_1 = D_2$, $n_1 = n_2$ to Eq. (2.4) gives:

$$\lambda_{02} = \lambda_{01}\sqrt{z_1 / z_2} \tag{3.1}$$

Note that Eq. (3.1) resembles other elementary equations: $q = \sqrt{r/\lambda} \cdot q_U$; rest mass $m_0 = \sqrt{F_{el}/F_U} \cdot m_p$, $F_{el} = q_U^2 / \lambda^2 4\pi\varepsilon_0$; $m_P = \sqrt{ch/G}$; $F_U = c^4/G$; λ = Compton wavelength.

The first information to emphasize is that the values λ_{02} calculated according to Eq. (3.1), that appears directly from the application of the model, transforms the value of the wavelength that relates the CMB λ_{01} with the cosmological red-shift, in values whose distribution coincides in a surprising way and with great accuracy, with the experimental distribution of wavelengths that shape the Cosmic Infrared Background (CIB) whose measurements are reflected in Figures 3.1 and 3.2, coinciding in turn with a wide conformity, the average value $\overline{\lambda}_{02}$ with the maximum experimental CIB value:

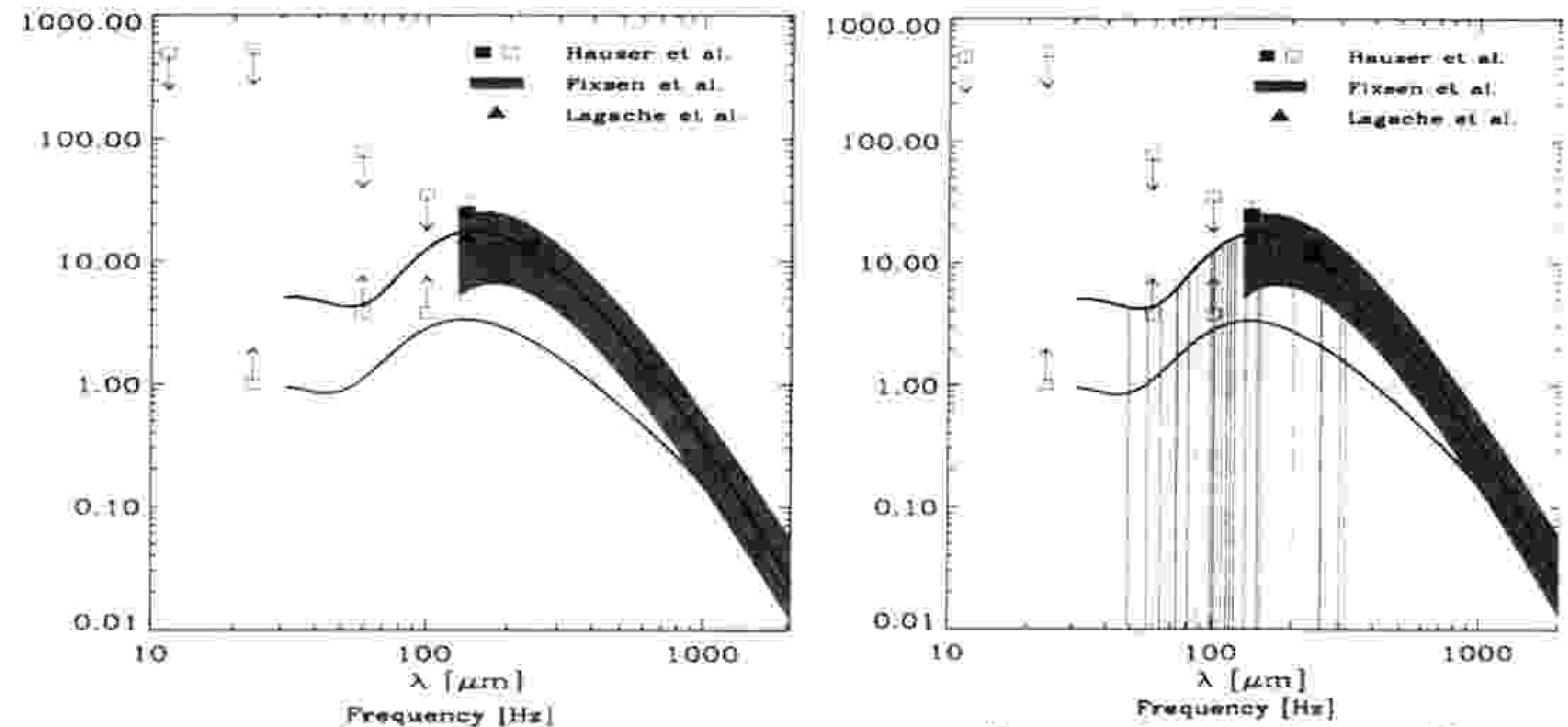

Figure 3.1. Cosmic Infrared Background spectrum. [1], [2], [4], [5]. The second graph represents the distribution of the wavelengths λ_{02} calculated from Eq. (3.1) and Table 3.1. The vertical dark gray line represents the average value, $\overline{\lambda}_{02} = \lambda_{max}$ CIB. A wide conformity is observed among the experimental values and the information contributed by the model and the considered hypothesis.

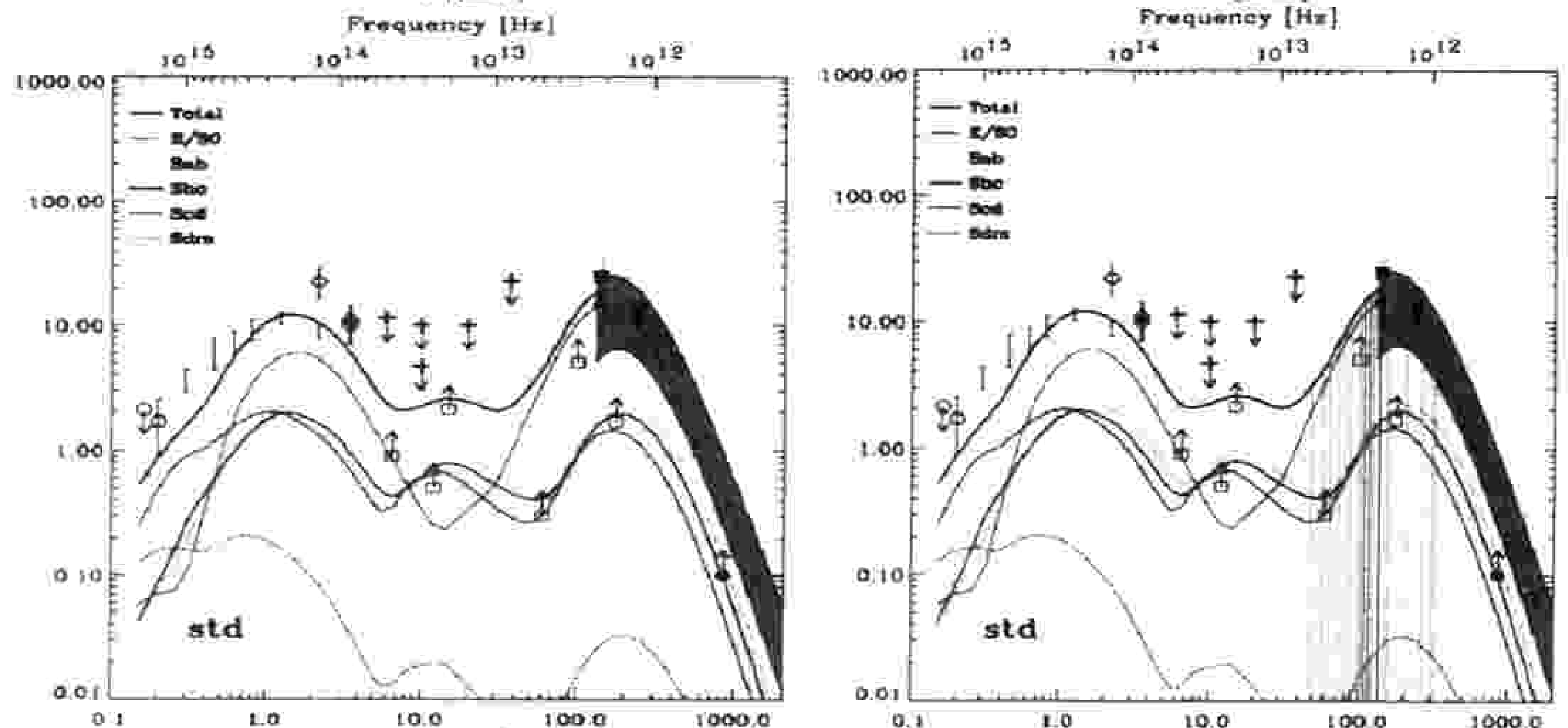

Figure 3.2. Cosmic Background spectrum from optical to millimeter wavelengths. A total conformity is observed among the experimental distribution and the distribution contributed by the model.

All vertical axes are νI_ν [nM/m^{2}sr].

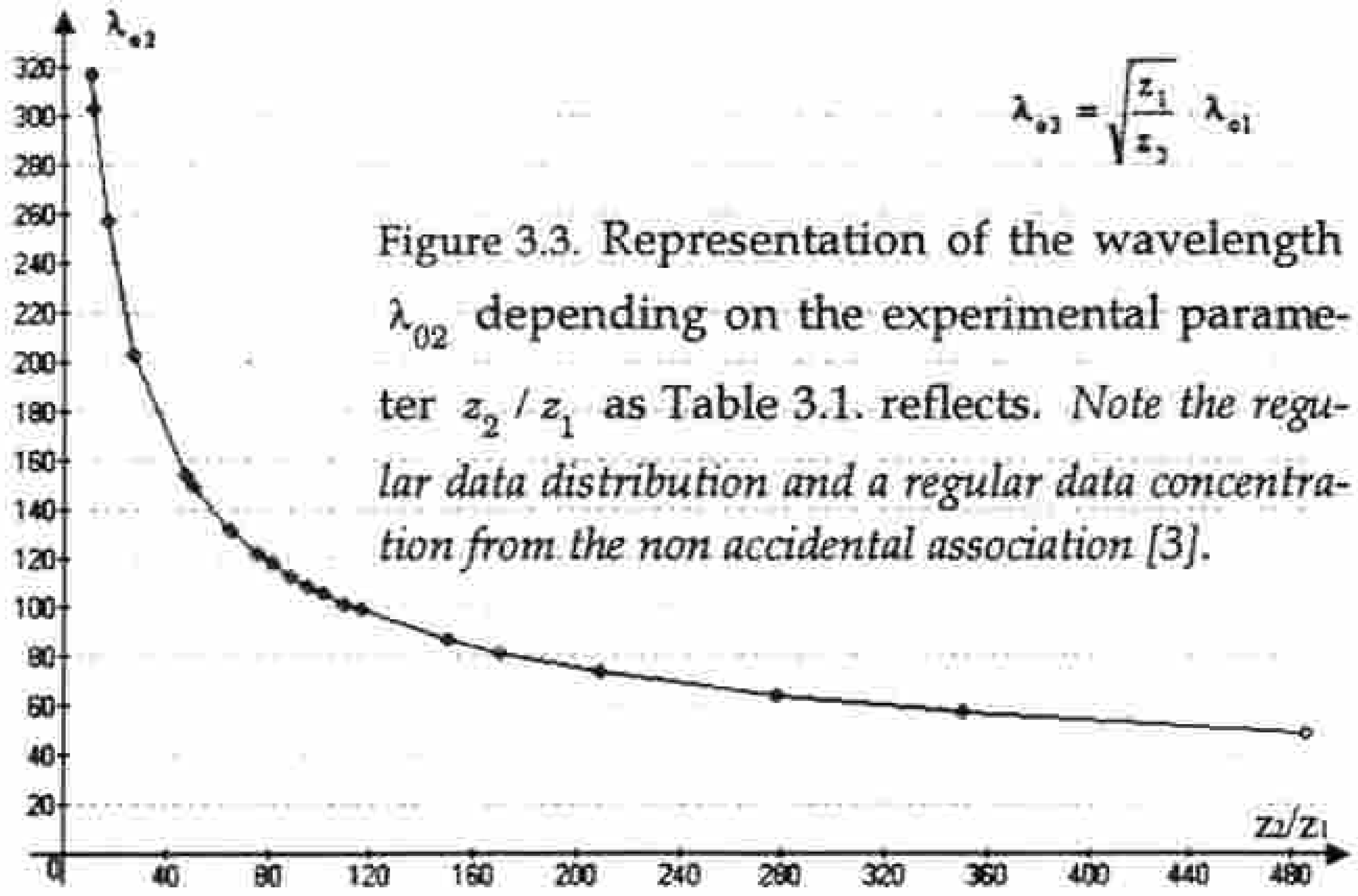

Figure 3.3. Representation of the wavelength λ_{02} depending on the experimental parameter z_2 / z_1 as Table 3.1. reflects. *Note the regular data distribution and a regular data concentration from the non accidental association [3].*

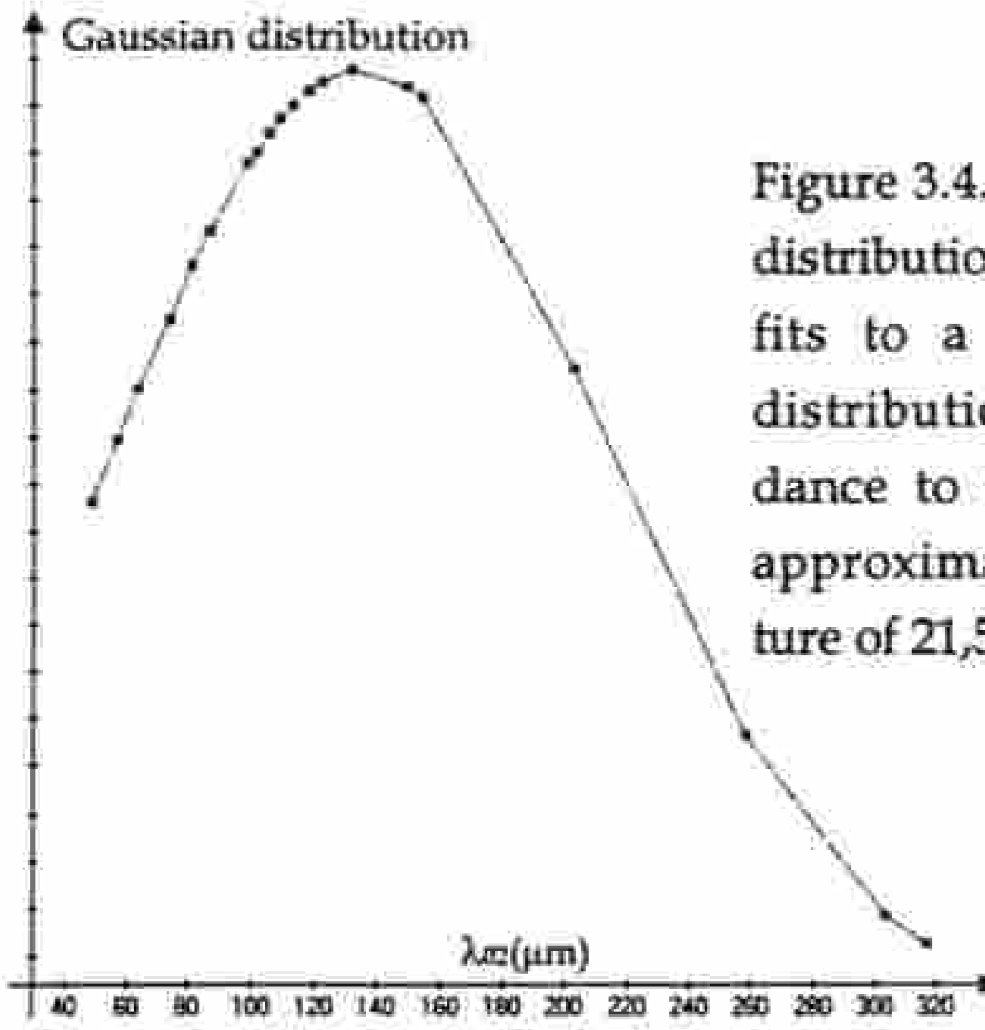

Figure 3.4. The normal distribution for λ_{02} data fits to a black body distribution in accordance to CIB with an approximate temperature of 21,5 °K.

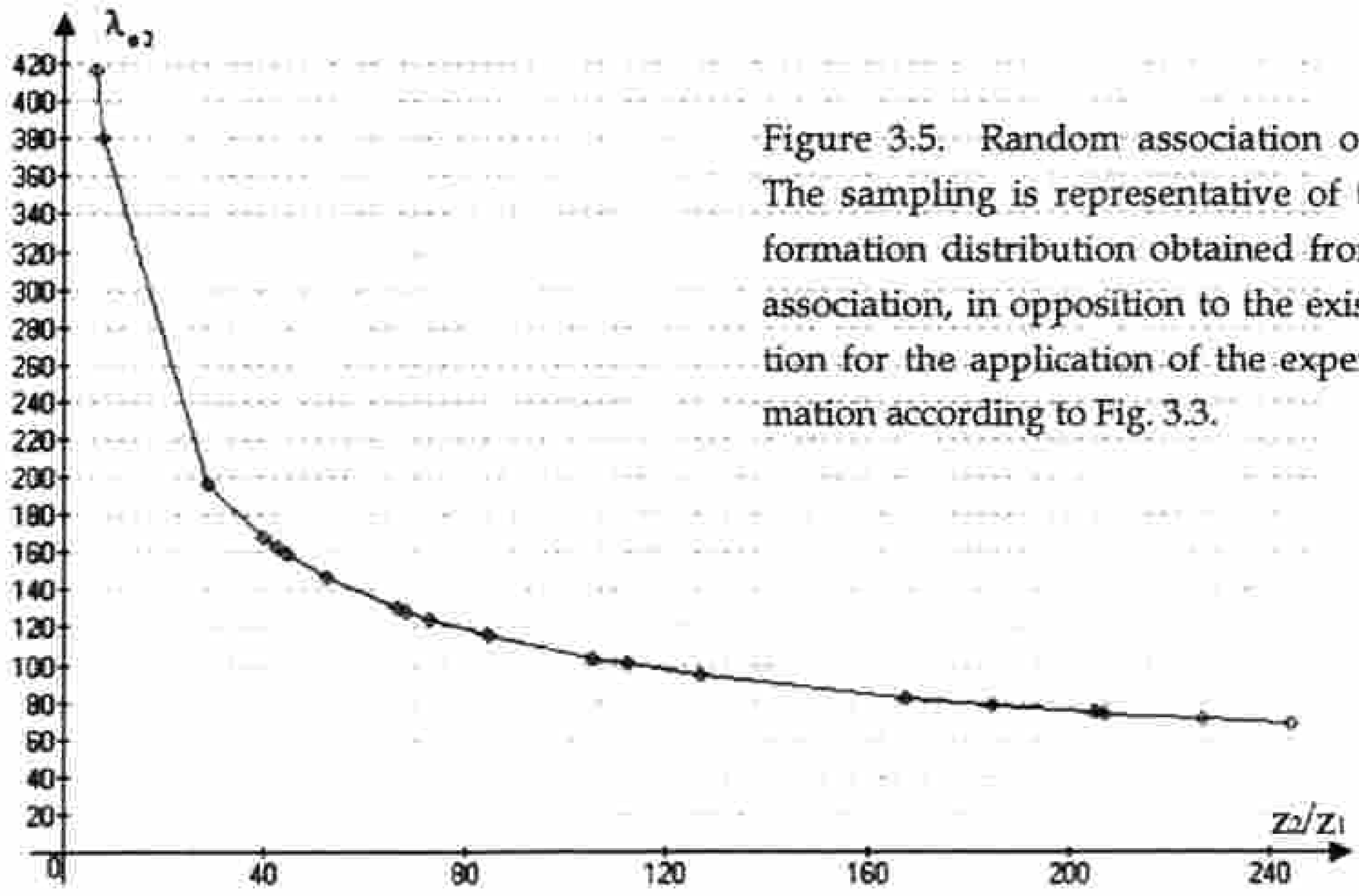

Figure 3.5. Random association of the samples. The sampling is representative of the chaotic information distribution obtained from the random association, in opposition to the existing classification for the application of the experimental information according to Fig. 3.3.

TABLE 3.2.	RANDOM ASSOCIATION	
	z1	z2
1	0,018	0,77
2		0,72
3		0,95
4		2,02
5	0,009	0,60
6		2,20
7		0,07
8		0,67
9	0,02	2,11
10		1,46
11	0,0057	1,17
12	0,018	1,53
13	0,0083	1,88
14	0,14	0,91
15	0,0667	1,94
16	0,0267	1,83
17	0,00758	1,57
18		0,96
19		0,34
20		1,40

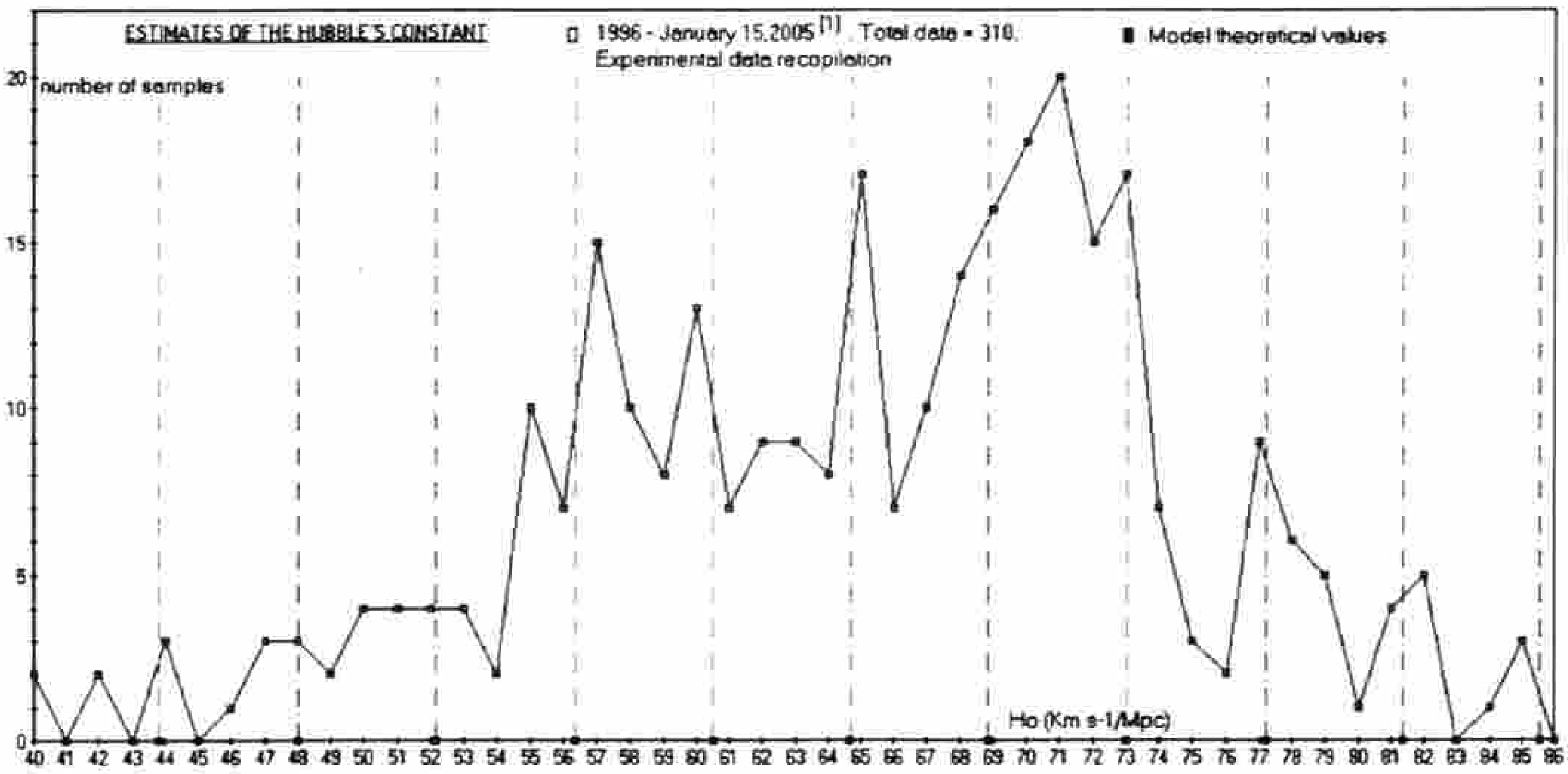

Figure 3.6. The number of samples obtained for H_0 values experimentally measured between year 1996 and the last update of the source[1], are represented on the axis of ordinates. Experimental values of the Hubble constant, in spite of experimental errors, agglutinate around the predicted ones by the model, forming domes around the values with peak points just in theoretical determinations or insinuating this statement. This representation will be indicative of any global pattern about Hubble constant behaviour, increasing accuracy by becoming larger the data accumulation despite the existence of inaccuracies according to experimental measurements. In fact, the existence of the anticipated pattern by the model is peeked on graph in which *Ho insinuates multiple possible quantized values separated 4,1 Km·s⁻¹/Mpc (see table), which closely coincides with the model predicted value 4,17 Km·s⁻¹/Mpc.*

STATISTICS for Ho (Km·s⁻¹/Mpc)					
Source: **Harvard-Smithsonian Center for Astrophysics. John Huchra.**					**Model**
	Peak points formation (Experimental Ho values)	Average	Peak points difference N, N-1		Ho values
1	43,44,45	44,0			43,8
2	45,46,47,48,49	47,0	3,0		48,0
3	49,50,51,52,53,54	51,5	4,5		52,2
4	54,55,56,57,58,59	56,5	5,0		56,3
5	59,60,61	60,0	3,5		60,5
6	61,62,63,64,65,66	63,5	3,5		64,7
7	66,67,68,69,70,71,72	69,0	5,5		68,9
8	72,73,74,75,76	74,0	5,0		73,0
9	76,77,78,79,80	78,0	4,0		77,2
10	80,81,82,83	81,5	3,5		81,4
11	83,84,85,86	84,5	3,0		85,6
			ΔH_0	4,1	

4. Conclusions and Interpretations

This Section relates some points that synthesize the information obtained from the rest of the paper. These follow:

- Every emission source (electron, galaxy,...) could imply an oscillator with an energy state $E_H = f(n,\nu_0) = hH_0$, generally with values $n \in [13,19]$ for cosmic objects, depending on the source.

- λ_0 is a wavelength that appears in the model to connect the intrinsic cosmic red-shift with cosmic backgrounds. It is a constant value during all the trajectory of the electromagnetic wave, but it depends on emission sources. The electromagnetic radiation for any interval transmits from the emission source along all the trajectory through the vacuum structure a constant value $\varepsilon_0 \frac{d}{dt}\oint_S \mathbf{E}\cdot d\mathbf{S} = q_U c/\lambda_0$ so that λ_0 is observed as cosmic backgrounds. An intrinsic cosmic red-shift appears from the vacuum parameters and from the electromagnetic waves transmission process.

- $H_0 = f(n,\lambda_0)$, and it is proportional to a vacuum constant: $V = (\Omega_0, x_p, c, q_U^2)/h$. From the consideration of the association that leads to the surprising relationship between the CIB and quasars, it seems that the distinctive determinant parameter would be λ_0. According to the application of the model, λ_0 *for galaxies* $\in$ *CMB*, $\lambda_0 = \lambda_{01} = \lambda_{\max}(CMB)$; λ_0 *for quasars* $\in$ *CIB*, $\lambda_0 = \lambda_{02} \in (CIB)$ *so that* $\bar{\lambda}_{02} = \lambda_{\max}(CIB)$ (See Figs. 3.1. and 3.2.), which imply $\nu_{\text{quasar}} = H_{0\ \text{quasar}} > \nu_{\text{galaxy}} = H_{0\ \text{galaxy}}$.

And as a general conclusion and interpretation, the CMB and CIB would be reinterpreted as the 'noise' produced by vibration states related to the cosmic objects included in the Universe.

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CORRESPONDENCE

Gravitational Red Shift isn't a Consequence of GRT

The usual deduction the gravitational frequency shift in general relativity theory (GRT) is based on invariance of the metric form $ds^2 = g_{ij}(\Phi)x^i x^j$ (see, *e.g.*, [1,2]). Here $i, j = 0,1,2,3$, g_{ij} is the metric tensor; Φ is the gravitational potential. But, as was mentioned earlier in [3], this is not a correct procedure.

Strictly speaking, the behavior of characteristics of an arbitrary wave process is defined by the 'wave interval' squared $\kappa^2 = g^{ij}(\Phi)k_i k_j$, where the wave 4-vector $k_i = (\nu, \mathbf{k}c)$, $k = 1/\lambda$. For example, the relation between values in the absence of and in the presence of a gravitational field is defined by the equation

$$\eta^{ij}K_i K_j = \kappa^2 = g^{ij}(\Phi)k_i k_j \quad (1)$$

where η^{ij} is Minkowski's tensor. Since for light $ds^2 = \kappa^2 = 0$, it follows that both the left and right sides of Eq. (1) are zero. And this means that, strictly speaking, there is no direct relation between the corresponding wave characteristics of light. In other words, *the known equation for the frequency* ν_g *of photon emitted in a gravitational field,*

$$\nu_g = \nu(1 + \Phi/c^2) \quad , \quad (2)$$

does not follow from GRT!

As was ascertained recently [4], Eq. (2) is the consequence of the energy conservation law as applied to the photon emission in a gravitational field

$$M^*c^2 + M^*\Phi = Mc^2 + M\Phi + h\nu_g \quad (3)$$

(Strel'tsov's equation; at one time the author underestimated the importance of this result). **(continued on p. 97)**